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Fractal Assessment of Crack Propagation in Cubic Concrete Specimens

Ali Fooladi [1], Mohammad Reza Banan [1*]
[1] Department of Civil and Environmental Engineering,
School of Engineering, Shiraz University, Shiraz, Fars 71348-51156, Iran
[1*] Corresponding Author, E mail: banan@shirazu.ac.ir

Abstract— Fractal patterns of crack propagation have been studied through an experimental-numerical research. Cubic specimens curing for 28 days with different compressive strength are tested under static loading. The specimens' surfaces have been processed undercoating and brushing white color. All the testing procedure is cinematize under perfect diffusion of light. The taken pictures have been divided into several snapshots and each individual snapshot has been image processed. Following this precedure a clear image of the crack propagation is obtained. The crack propagation patterns have been then fractal analyzed. The box counting fractal analysis is utilized over the processed images and the fractal dimension of the arborous crack patterns is calculated. As the results depicts the saturated fractal dimension increases as the compressive strength of the specimen increases.

Key words: *Crack propagation, Fractal analysis, Fractal pattern, Box counting*

Introduction

Identifying cracks induced by loading and preserving these stress-induced micro cracks are key to understand the mechanism of the crack formation and propagation [1]. Damage and fracture characterizing the compressive failure of heterogeneous materials such as rocks and concrete are complex processes involving wide ranges of time and length scales, from the micro- to the structural scale. They are governed by the nucleation, growth and coalescence of micro cracks and defects, eventually leading to the final collapse. The collapse causes the loss of the classical mechanical parameters, such as nominal strength, dissipated energy density and deformation at failure, as material properties [2]. Several experimental confirmations are received about the hypothesis of damage domains showing fractal patterns in concrete structures [3]. The fundamental nature of scaling laws for both strength and toughness of concrete elements is deeply connected with fractal statistical aspects [4]. Applying fractal analysis in crack propagation studies is important because of the concrete elements behavior under loading is a complex function of different controlling factors and their sensitiveness to the initial conditions [5]. This fact means that the crack propagation exactly fits the nonlinearity frameworks and needs nonlinear instruments of study in regards.

Most researches have been concentrated on the failure mechanism and the crack propagation procedure not on the crack patterns and geometry. The main objective of this paper is to investigate the fractal patterns of cracks in concrete specimens.

The ingredients, mechanical properties and the mix design of the concrete upon the compression strength are as follows:

The materials used are the Portland cement type 1, gravel, sand which are prepared from local suppliers and drinking water. The results of grain size analyze for gravel and sand are summarized in Table 1 and 2. Other required parameters of sand and gravel for mix design are depicted in Table 3. The mix design of the concrete specimens for $f_c^{'} = 30\ mpa$. $f_c^{'} = 35\ mpa$ and $f_c^{'} = 40\ mpa$ are calculated according to ACI-211-.1-91 standard and shown in Table 4.

Running the experiments begins with making the cubic concrete specimens according to the identified mix designs under controlled laboratory conditions and then curing the specimens for 4 weeks. All cubic specimens are surface processed with undercoating and brushing white color which improves revealing the cracks under pressure (different methods and techniques have been applied by researchers to view the cracks and micro cracks in concrete elements [6]). The specimens are then tested using compression test machine with loading rate of 2 $kN/\sec$ while the appropriate light is supplied and all the steps are recorded by the camera. Then the pictures are transformed into several snapshots (we made each specimen's compressive strength test into 16 snapshots). Each snapshot is image processed and the crack pattern is revealed.

Image processing procedure helps us to capture just the crack expansion image from the whole background images. The processed crack images are then fractal analyzed and fractal dimensions are extracted. For each specimen the obtained fractal dimensions vs. the snapshot's number are drawn and the developed graph shows the variations of the fractal dimension with increasing the compression force and the crack growth.

The philosophy of fractal concept is to reach a constant fractal dimension in which any load increase and the crack expansion does not change this value. This number is called the saturated fractal dimension.

Table 1: Grain size analysis for the coarse aggregates

Passing percentage	Remaining percentage	Remaining weight (gr)	Sieve No.
100.0	0	0	$1\frac{1}{2}$
97.9	2.1	478	1
75.2	22.8	5151	$\frac{3}{4}$
31.0	44.2	10015	$\frac{1}{2}$
11.6	19.5	4414	$\frac{3}{8}$
0.6	11.1	2495	4
0	0.6	117	Pan

Table 2: Grain size analysis for the fine aggregates

Passing percentage	Remaining percentage	Remaining weight (gr)	Sieve No.
96.4	26.0	18	4
62.4	24.0	170	8
50.0	12.4	62	16
18.4	216.0	158	30
118	66.0	22	50
5.2	66.0	22	100
28.0	2.4	12	200
0	28.0	14	Pan

Fractal Analysis

Fractal geometry is rooted in the works of late 19th and early 20th century mathematicians who found their desire in generating complex geometrical structures from simple objects like a line, a triangle, a square, or a cube as the initiator by applying a simple rule of transformation (the generator) in an infinite number of iterative steps[7]. The complex structure that resulted from this iterative process proved equally rich in detail at every scale of observation, and when their pieces were compared to larger pieces or to those of the whole, they proved similar to each other [8].

The concept of introducing a fractal dimension to describe a structure, which look the same for all length scales, was first proposed by Mandelbrot [9]. Although in strict terms, this is a purely mathematical concept, there are many examples in nature that closely approximate a fractal object, though only over particular ranges of scale. Such objects are usually referred to as self-similar to indicate their scale-invariant structure. In simple terms, the common characteristic of such fractal objects is that their length (if the object is a curve, otherwise it could be the area or volume) depends on the length scale used to measure it, and the fractal dimension tells us the precise nature of this dependence [10]. Such objects (processes) have details at arbitrarily small scales, making them too complex to be represented by Euclidean space [11].

Natural processes are usually characterized by a complex pattern of correlations that appears following multiple nested spatial scales. Fractal analysis can be employed to better understand the nature of these fluctuations and to disclose the inherent dependency of data in natural fractal structures 11. To measure geometric (fractal) scaling of an object, two indices are commonly employed. The first one is fractal dimension, denoted by D and the second one is Hurst exponent, H. It is notable that the two are two sides of a coin. Having knowledge of the value of Hurst exponent, one can determine the fractal dimension and vice versa.

Box counting method

The box counting algorithm is instinctive and easy to apply [12]. A fractal curve is a curve of infinite details by virtue of its self similarity. The length of the fractal curve indefinitely increases with increasing the resolution of the measuring mechanism. The fractal dimension determines the increase in detail, and therefore length, at each resolution step. For a fractal, the length L as a function of the resolution of the measurement tool δ is:

$$L(\delta) \propto \delta^{-D} \quad (1)$$

And D is the fractal dimension. Box counting algorithms measures $N(\delta)$ for varying δ by counting the number of non overlapping boxes of size δ, needed to cover the curve. Fractal dimension (D) is obtained from the following equation:

$$D = -\lim_{\delta \to 0} \frac{\log N(\delta)}{\log(\delta)} \quad (2)$$

Where $N(\delta)$ corresponds to the number of boxes required to completely cover the curve. Figure 1 shows the mechanism of boxes covering a curve which can be an object or a time series.

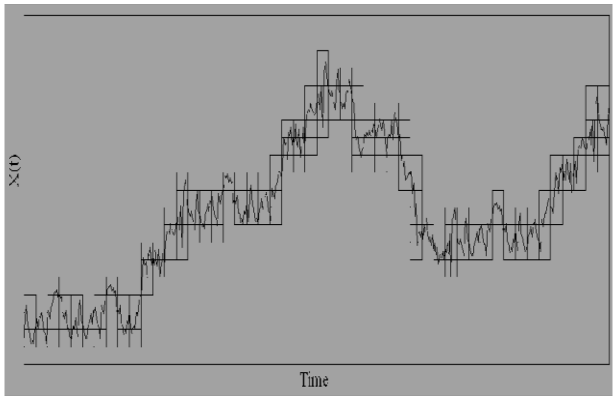

Fig. 1 Box counting method calculates the fractal dimension [8]

RESULTS

According to the fractal definition a fractal pattern is scale independent. This concept can be traced in crack propagation mechanism. Fractality plays an important role in crack propagation, and also friction phenomena are strongly affected by fractal contact mechanisms [13]. Cracks propagation starts from tiny and narrow line on the cubic face to a tree shaped expanded figure. In this case the fractal dimension varies theoretically between 1 and 2. The more the specimen is loaded and the crack propagates the fractal dimension increase towards 2. The idea is that this figure object would get a fractal pattern once and the fractal dimension will saturate at the time so that more expansion of crack would not change the fractal dimension. The same incident has happened for the specimens tested. Below is the results yielded for the specimens with different Compressive strength.

Fig. 2 shows the result of the fractal dimension for specimens with $f_c^{'} = 30\ mpa$.

and continuous grading. Fig 3 depicts the result of fractal dimension and the crack propagation image for each snapshot regarded to the specimens with $f_c^{'} = 30\ mpa$.

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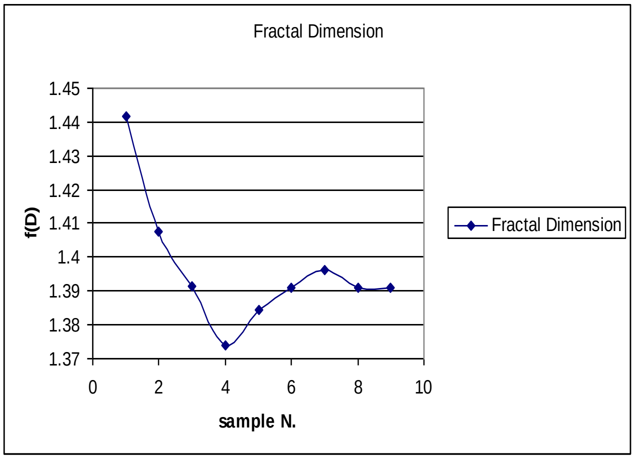


Fig. 3 Fractal dimension Vs. specimen's number for

$$f_c^{'} = 30 \ mpa$$

As can be seen in Fig. 3 fractal dimension is saturated on 1.39 for the analyzed snapshots of concrete specimens with $f_c^{'} = 30 \ mpa$. Figure 4 depicts the same manner for the specimens with $f_c^{'} = 35 \ mpa$. The fractal dimension is saturated on 1.3 which is smaller than 1.39. This decrease was predictable as the increase in the compressive strength may cause less damage on the cube surface which is revealed as less crack propagation and therefore lessen of fractal dimension. Figure 5 also shows the fractal dimension for specimens with $f_c^{'} = 40 \ mpa$ which is saturated at 1.4. Although the mechanism of decreasing the fractal dimension Vs. increasing the compressive strength seems not valid anymore but fractal dimension has saturated to a very close value again which depicts the fractal nature of the crack propagation in these specimens as well.

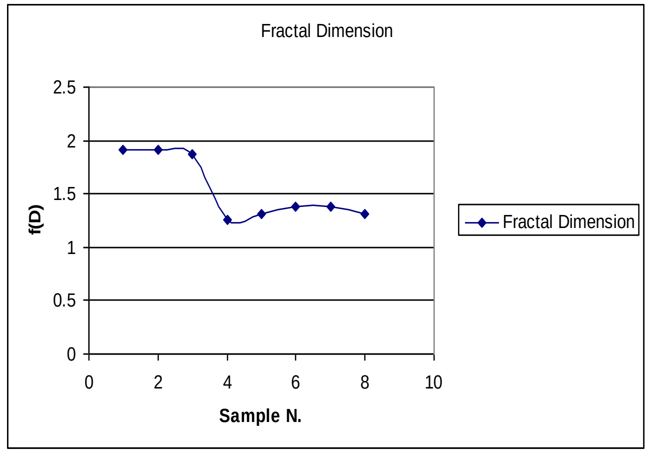


Fig 4 Fractal dimension Vs. specimen's number for $f_c^{'} = 35$ mpa

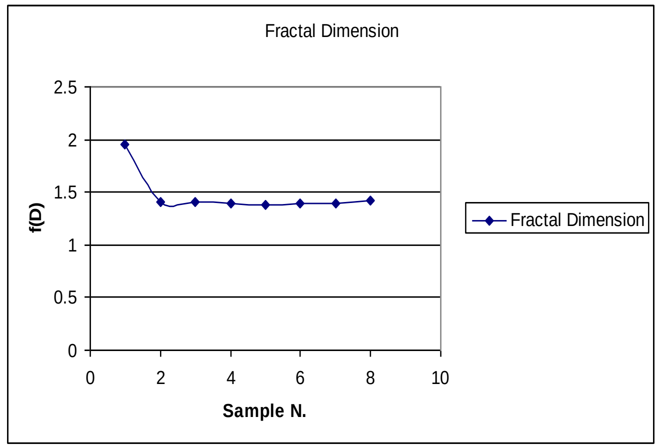


Fig. 5 Fractal dimension vs. specimen's number for $f_c^{'} = 40$ mpa

CONCLUSION

Results of the fractal analysis of the crack propagation in cubic concrete specimens with the compressive strength 30, 35 and 40 mpa are summarized as follows:

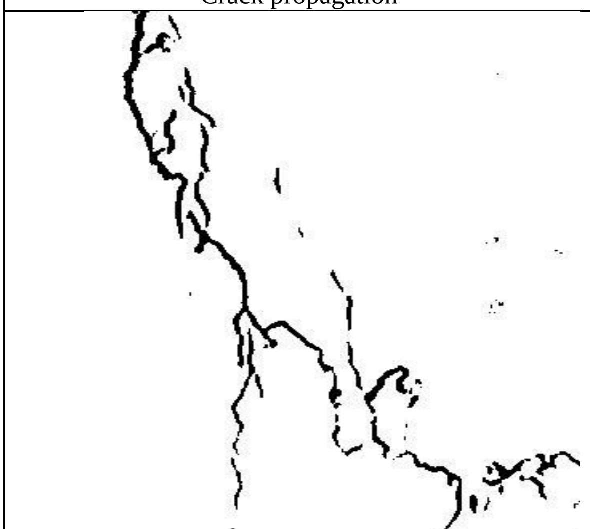
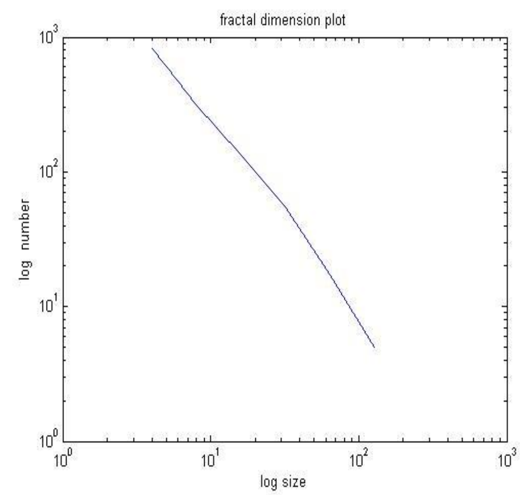
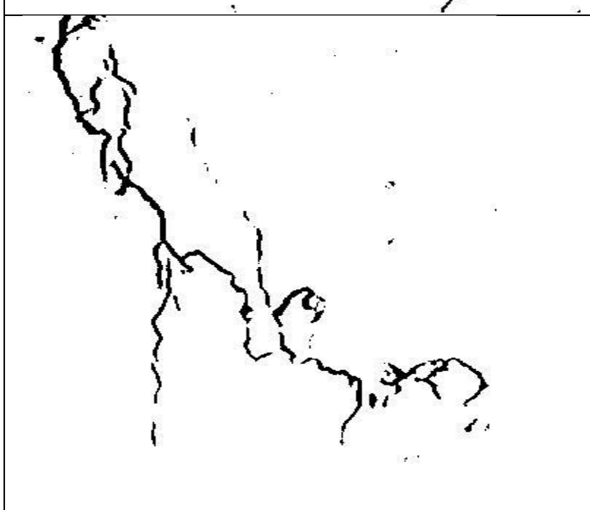
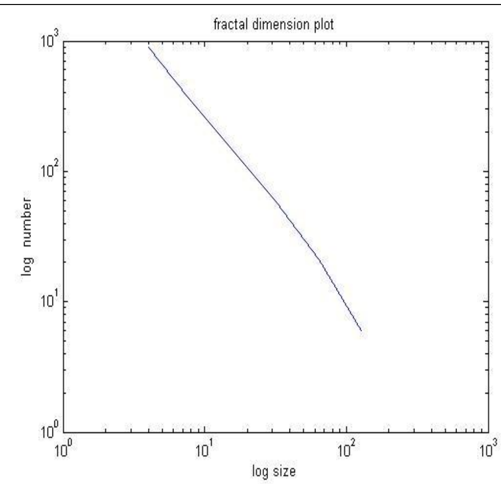
Fractal analysis yields the fractal dimension of each snapshot. Concrete specimen with continuous grading shows fractal pattern propagation of cracks. Uniform distribution of stress due to continuous grading has prevented establishing micro fracture mechanisms. Crack propagation has followed and evolves into a tree shaped pattern. Fractal dimension of crack propagation gradually increase and finally saturates an identified value which represent the evolution of their fractal pattern. The saturated fractal dimensions for continuous grading specimens with compressive strength 30, 35 and 40 *mpa* are 1.3, 1.39 and 1.4.

Table 3: Some important parameters for mix design calculations

Fineness module	Specific weight	D_{max}	Density kg/m^3	Absorption ratio %	Specific mass (DRY)	Specific mass (SSD)	Moisture %	
---	2.717	25 mm	1414	2.087%	2.571	2.656	1.88%	Gravel
3.56	3.155	---	1567	2.25%	2.945	3.01	12.8%	Sand

Table 4: Concrete ingredients (per kg) according to the ACI-211.1-91 mix design

Water	Cement	Sand	Gravel	Compressive strength
223.5	357.4	1025.4	857.0	*mpa* $f_c = 30$
2.021	410.0	834.3	950.5	*mpa* $f_c = 35$
221.5	459.5	922.1	916.6	*mpa* $f_c = 40$

Crack propagation	Fractal dimension
	
	

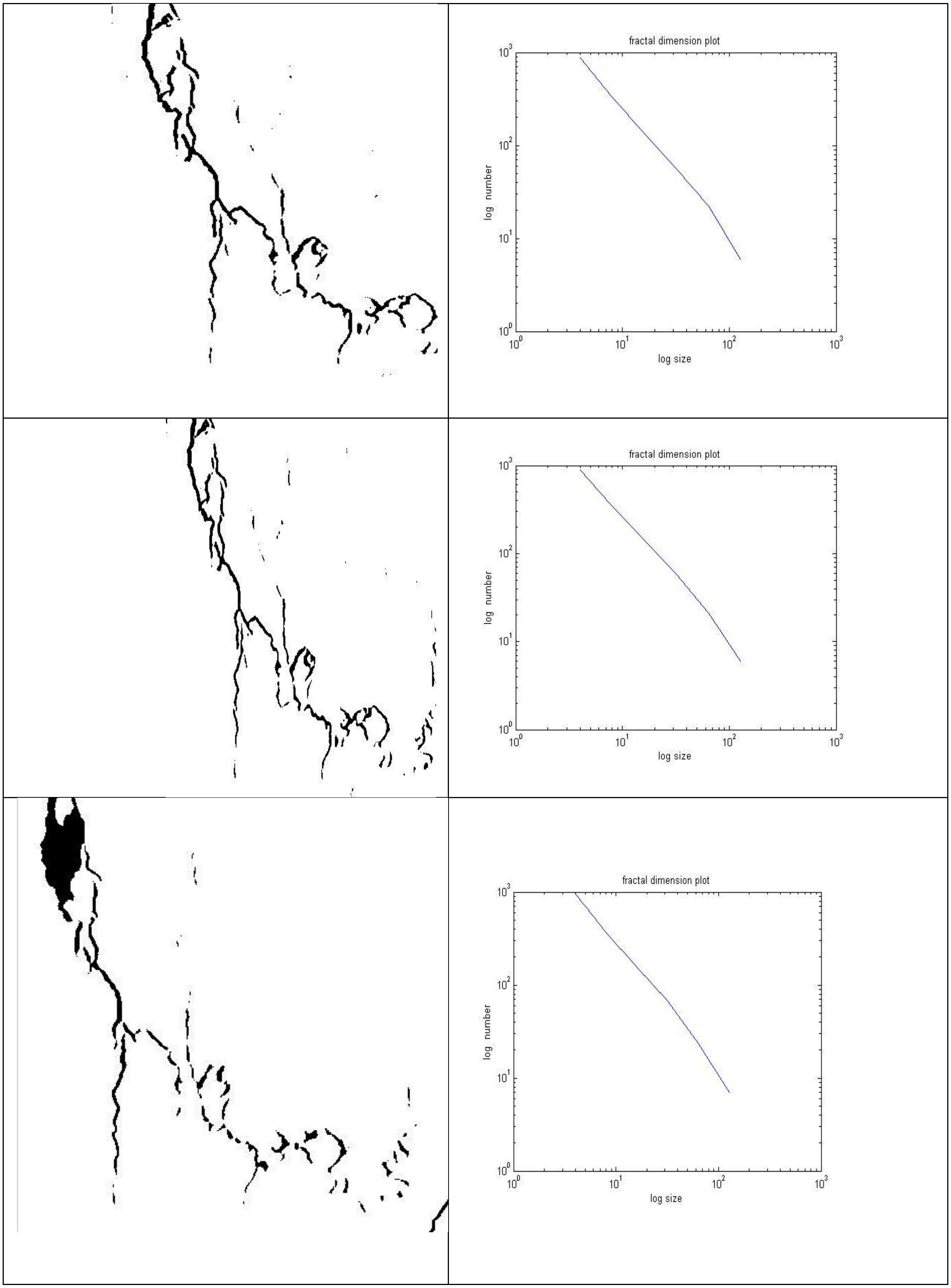


Fig. 2 Fractal dimension for specimens with $f_c^{'} = 30 \;\; mpa$ and continuous grading and the crack propagation pattern

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