

Convective Heat and Mass Transfer Flow of Viscous Fluid with Variable Viscosity in the Presence of Thermophoresis Particle Deposition

B. ALIVENE¹, Dr.M.SREEVANI²

¹Research Scholar, Department of Mathematics, Rayalaseema University, Kurnool-518007(AP), India.

²Department of Mathematics, S.K.U. College of Engineering & Technology, Sri krishnadevaraya University, Anantapuramu – 515003 (AP), India.

Abstract: In this paper, we investigate the combined influence of thermal radiation, Soret and Dufour effect on free convective heat and mass transfer flow of a viscous incompressible fluid in the presence of suction/injection and thermophoresis deposition particle. The non-linear equations governing the momentum, temperature and concentration along with the boundary conditions are solved by using finite element method with Mathematica 6.1 software.

Introduction

Thermophoresis is the term describing the fact that small micron sized particles suspended in a non-isothermal gas will acquire a velocity in the direction of decreasing temperature. The gas molecules coming from the cold side have a lesser velocity than those coming from the hot side of the particles. The faster moving particles collide with the particles more forcefully. This difference in momentum leads to the particle developing a velocity in the direction of the cooler temperature. The velocity gained by the particles is defined as the thermophoretic velocity and the force experienced by the suspended particles due to the temperature gradient is known as the thermophoretic force. Corrosion of heat exchanger, fouling of gas turbine blades, the blackening of glass globe of kerosene lanterns, chimneys and industrial furnace walls by carbon particles are some examples of thermophoresis. In manufacturing graded index silicon dioxide and germanium dioxide optical fibre performs used in the field of communications have used the principles of thermophoresis. The thermophoresis in laminar flow over a flat plate for cold and hot plate conditions has been analysed by Goren [10]. The effect of heat generation /absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat

surface has been studied by Chamkha et al (6). The natural convection over a vertical flat plate in a porous medium with thermophoresis has been discussed by Chamkha and Pop [7]. The effect of variable viscosity and thermophoresis on a boundary layer flow with chemical reaction has been studied numerically by Seddeek [18]. The effects of heat generation or absorption on thermophoresis free convection boundary layer from a vertical flat plate embedded in a porous medium have been discussed by Chamkha et al (8). The effects of Soret and Dufour with thermophoresis in a non-Darcy porous medium in the presence of suction/injection has been discussed by Partha [16]. The effects thermophoresis, Soret and Dufour are very significant for the fluids which has higher temperatures and concentration gradient. The thermal-diffusion (Soret) effect is corresponds to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient and the diffusion-thermo (Dufour) effect corresponds to the heat flux produced by a concentration gradient. Usually, in heat and mass transfer problems the variation of density with temperature and concentration give rise to a combined buoyancy force under natural convection and hence the temperature and concentration will influence the diffusion and energy of the species. Several authors (1,2,5,9,12,15,16,17) have investigated Soret and Dufour effects in different configuration under varied conditions. The effect of radiation on Mhd flow and heat transfer problem have become more important industrially. At high temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent

equipment. Several researches (3,4,11,14,17) have discussed effect of thermal radiation on heat and mass transfer flow.

5.2. MATHEMATICAL FORMULATION:

We discuss the two-dimensional steady mixed convective heat and mass transfer flow of a viscous incompressible, non-conducting, fluid through a porous medium in the presence of thermophoresis. We consider a rectangular cartesian coordinate system $O(x,y)$ with x -axis along the vertical surface and the y -axis perpendicular to plate. The vertical surface and the fluid are maintained at same temperature and concentration initially. Instantaneously they raised to a temperature $T_w (> T_\infty)$ and concentration $C_w (> C_\infty)$ which remain unchanged. To help the understanding of the mass deposition variation on the surface the effects of thermophoresis are is considered in the diffusion equation. We assume that the temperature gradient in the y -direction is much larger than that in x -direction and therefore thermophoretic velocity component normal to the surface is of more importance.

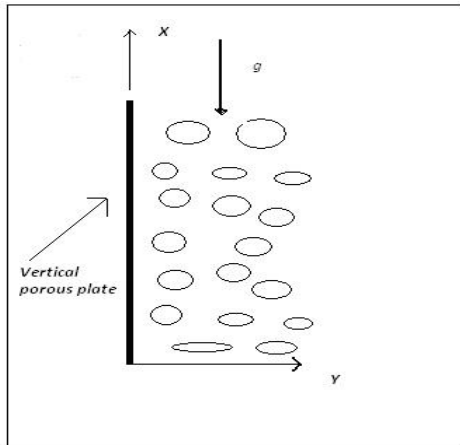


Fig 1: Physical Model of the Coordinate System

Under the above assumptions, the equations governing the flow, heat and mass transfer with Boussinesq approximation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \beta g (T - T_\infty) + \beta^* g (C - C_\infty) - \frac{\nu}{k} u - \frac{\sigma \mu_e^2 H_0^2}{\rho_o} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_f}{C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{1}{C_p} q'' + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} - \frac{1}{C_p} \frac{\partial (q_R)}{\partial y} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_c (C - C_\infty) + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - \frac{\partial (V_T C)}{\partial y} \tag{4}$$

The relevant boundary conditions are

$$\begin{aligned} u = ax, v = v_w(x), T = T_w, C = C_w & \quad \text{at } y = L \\ u = 0, v = 0, T = T_\infty, C = C_\infty & \quad \text{as } y \rightarrow \infty \end{aligned} \tag{5}$$

The thermophoretic velocity V_T , which appears in Eq.(4), is expressed as

$$V_T = - \frac{k_1 \nu}{T_r} \frac{\partial T}{\partial y} \tag{6}$$

in which k_1 is the thermophoretic coefficient and T_r is the reference temperature. A thermophoretic parameter τ is given by the relation

$$\tau = - \frac{k_f (T_w - T_\infty)}{T_r} \tag{7}$$

Where the typical values of τ are 0.01, 0.1 and 1.0 corresponding to approximate

Values of $-kl(T_w - T)$ equal to 3, 30, 300K for a reference temperature of $T = 300K$.

Using Roseland approximation the energy equation (3) with non-uniform heat source reduces to

$$\begin{aligned} \rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_f \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{k_f U_0}{\nu x} (A_1 (T_w - T_\infty) u + B_1 (T - T_\infty)) + \\ + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} + \frac{16 \sigma^* T_\infty^3}{3 \beta_r} \frac{\partial^2 T}{\partial y^2} \end{aligned} \tag{8}$$

Introducing the following similarity transformation

$$\begin{aligned} \eta = \sqrt{\frac{U_0}{2\nu x}} y, \quad u = U_0 f'(\eta), \quad v = - \left(\frac{\nu U_0}{2x} \right)^{1/2} (f(\eta) + \eta f'(\eta)), \\ \theta = \frac{T - T}{T_w - T}, \quad \phi = \frac{C - C}{C_w - C} \end{aligned}$$

(9)

the equations(2),(4) and (9) reduce

$$f''' + ff'' + G_r (\theta + N\phi) - \left(\frac{D^{-1}}{R_e} + M^2 \right) f' = 0 \tag{10}$$

$$\theta'' + Pr f \theta' + Pr Du \phi'' + \frac{4Rd}{3} \theta'' + Pr (A_1 f' + B_1 \theta) + Pr Ec (f'')^2 = 0 \tag{11}$$

$$\phi'' + Sc f \phi' + Sc Sr \theta'' - \tau (\theta' \phi' + \theta'' \phi) = 0 \tag{12}$$

The transformed boundary conditions are

$$f'(0)=1, f(0)=V_0\sqrt{\frac{2x}{\nu U_0}}, \theta(0)=1, \phi(0)=1 \text{ at } \eta=1 \quad (13)$$

$$f'(\infty)=0, \theta(\infty)=0, \phi(\infty)=0 \text{ as } \eta \rightarrow \infty$$

THE METHOD OF SOLUTION

The non-linear, coupled equations governing the flow, heat and mass transfer have been solved numerically by using finite element technique. The variational form associated with the equations (11)-(13) over a typical two noded line at element

(η_e, η_{e+1}) is given by

$$\int_{\eta_e}^{\eta_{e+1}} w_1(f' - h)d\eta = 0 \quad (14)$$

$$\int_{\eta_e}^{\eta_{e+1}} w_2(h'' + fh' + G(\theta + N\phi) - (\frac{D^{-1} + M^2}{R_e})(h))d\eta = 0 \quad (15)$$

$$\int_{\eta_e}^{\eta_{e+1}} w_3((1 + \frac{4Rd}{3})\theta'' + P_r(f\theta' + Du\phi'' + (A1h + B1\theta)))d\eta = 0 \quad (16)$$

$$\int_{\eta_e}^{\eta_{e+1}} w_4(\phi\phi + Sc(\phi'f - \gamma\phi)ScSo\theta'' + \tau(\theta'\phi' + \theta''\phi))d\eta = 0 \quad (17)$$

where w_1, w_2, w_3, w_4 are arbitrary test functions and may be regarded as the variations in f, h, θ and ϕ respectively. The finite element method may be obtained from (14)-(17) by substituting finite element approximations of the form

$$f = \sum_{k=1}^3 f_k \psi_k, h = \sum_{k=1}^3 h_k \psi_k, \theta = \sum_{k=1}^3 \theta_k \psi_k, \phi = \sum_{k=1}^3 \phi_k \psi_k \quad (18)$$

We take $w_1=w_2=w_3=w_4= \psi_i^j (i, j = 1,2,3)$.

Substituting (18) and evaluating the integrals we get 3x3 local stiffness matrices. These matrices are assembled into a global matrix using interelement continuity and equilibrium conditions, boundary conditions. The ultimate coupled global matrices are solved to determine the unknown global values of velocity, temperature and concentration in the fluid region. In solving these matrices an iteration procedure has been adopted. The iteration is repeated until the condition $|u_{i+1} - u_i| < 10^{-6}$ is satisfied. The major physical quantities of interest in this problem are the local skin friction coefficient C_f , the local Nusselt number Nu_x and the local Sherwood number Sh_x , are defined, respectively, by:

$$C_f = \frac{f''(0)}{Re_x^{1/2}}, Nu_x = -\frac{\theta'(0)}{Re_x^{1/2}}, Sh_x = -\frac{\phi(0)}{Re_x^{1/2}}$$

The correctness of the current numerical method is checked with the results obtained by Loganathan et al [13] for the local skin friction coefficient C_f , local Nusselt number Nu_x and local Sherwood number Sh_x for various values of thermophoretic parameter τ and suction parameter V_0 . Thus, it is seen from Table 1 that the numerical results are in close agreement with those published previously.

COMPARISION:

The results of C_f, Nu_x, Sh_x with previously published data, $Du=0, Sr=0, Ec=0, A1=0, B1=0, Rd=0$

Table.1

Du	Sr	Nu_x	Sh_x
0.5	0.5	-0.952739	-0.514925
	1.0	-0.981055	-0.439975
	1.5	-1.01065	-0.359391
	2.0	-1.04169	-0.2727
	2.5	-1.07441	-0.179333
1	0.5	-0.870524	-0.521268
	1.0	-0.915672	-0.452475
	1.5	-0.966592	-0.373596
	2.0	-1.02494	-0.282221
	2.5	-1.0931	-0.174893
1.5	0.5	-0.786301	-0.527484
	1.0	-0.844034	-0.465539
	1.5	-0.914626	-0.389352
	2.0	-1.00453	-0.292617
	2.5	-1.12568	-0.163768
2	0.5	-0.700188	-0.53357
	1.0	-0.765569	-0.479223
	1.5	-0.852782	-0.407087
	2.0	-0.979596	-0.304151
	2.5	-1.19254	-0.13677
2.5	0.5	-0.612292	-0.539526
	1.0	-0.679561	-0.49361
	1.5	-0.778022	-0.427486
	2.0	-0.94833	-0.317407
	2.5	-1.481717	-0.141578

Table : 2

Parameter		Loganathan(13)		
		C_f	Nu_x	Sh_x
τ	1.0	6.034338	-2.580389	-5.260410
	2.0	6.034184	-2.580390	-5.804620
	3.0	6.034067	-2.580391	-8.366696
V_0	0.5			
	1.0			
	1.5			
	4.0	7.899134	-7.543495	-4.788675
	5.0	9.665340	-9.336595	-5.882326
	6.0	11.45915	-11.14177	-6.988321
Parameter		Present results		
		C_f	Nu_x	Sh_x

τ	1.0	6.034336	-2.580385	-5.260409
	2.0	6.034189	-2.580385	-5.804615
	3.0	6.034069	-2.580383	-8.366989
V_0	0.5	4.125482	-5.026587	-1.236587
	1.0	5.354135	-6.154873	-2.365989
	1.5	6.054877	-6.998514	-3.654871
	4.0	7.899139	-7.543493	-4.788673
	5.0	9.665336	-9.336592	-5.882322
	6.0	11.45919	-11.14169	-6.988319

Computed values of local Nusselt number and local Sherwood number for $V_0=0.5$; $Pr=0.71$; $Sc=0.22$; $Gr=5$; $Gm=5$; $Da=0.01$; $R=10$; $Ec=0.01$; $\phi=0.1$; $s=0.1$; $G1=0.5$; $B1=0.5$; $\phi=0.1$

DISCUSSION OF THE RESULTS

In this analysis we discuss the effect of thermal radiation, dissipation, Soret and Dufour on convective heat and mass transfer flow of a viscous, electrically conduction fluid past a stretching sheet in the presence of non-uniform heat source. The results are presented graphically in figures.2-19 for different parametric variations. Comparison of the present results with previously works are performed and excellent agreements have been obtained. The non-linear coupled differential equations are solved by Galerkin finite analysis with three nodded line segments. In the absence of the results are compared with Loganatham(13). Figs.2-4 represents the velocity, temperature and concentration with buoyancy ratio (N). It is found that when the molecular buoyancy force dominates over the thermal buoyancy force the velocity experiences an increase in the flow region irrespective of the directions of the buoyancy forces. The temperature and concentration reduces with increase in N irrespective of the directions of the buoyancy forces (figs 3 & 4). The skin friction and Sherwood number at the wall increases with N irrespective of the directions of the buoyancy forces while the Nusselt number increases with $N>0$ and depreciates with $N<0$ (table.3) Figs.5-7 depicts the influence of Radiation parameter on the velocity, temperature and concentration. It is observed that there is a significance rise in the velocity in the presence of thermal radiation throughout the boundary layer. Further increase in the values of thermal radiation parameter results in the increase of the boundary layer. The presence of the thermal radiation is very significant on the variation of temperature. It is seen that the temperature rapidly increases in

the presence of thermal radiation parameter throughout the thermal boundary layer. This may be attributed to the fact as the Rosseland radiative absorption parameter R^* diminishes the corresponding heat flux diverges and thus rises the rate of radiative heat transfer to the fluid causing a rise in the temperature of the fluid. The thickness of the boundary layer also increases in the presence of R_d . The effect of R_d on concentration is to diminish it in the solutal boundary layer. The skin friction and the Sherwood number at the wall enhances and the Nusselt number reduces with increase in R_d (table.3) Figs.8-10 represent the variation of velocity, temperature and concentration with space dependent heat source/temperature dependent heat source in the boundary layer. An increase in the space dependent/temperature dependent heat source increases the velocity and temperature in the boundary layer. This may be attributed to the fact that the presence of the heat source generates energy in the boundary layer and as a consequence the velocity and temperature rises. In the case of heat absorption $A<0, B<0$, the velocity and temperature falls, owing to the absorption of energy in the boundary layer. In the case on concentration we find that it reduces with $A>0, B>0$ and increases with $A<0, B<0$ in the boundary layer. The skin friction and the Sherwood number enhance while the Nusselt number reduces at $\phi=0$ with increase in $A_{11}>, B_{11}>0$. An increase in $A_{11}<0, B_{11}<0$ the Skin friction, Sherwood number reduces, the Nusselt number enhances at the wall (table.3) Figs.11-13 shows the influence of dissipation on the velocity, temperature and concentration. It is pointed out that the presence of Eckert number increases the velocity and temperature. This is due to the fact that the thermal energy is reserved in the fluid on account of friction heating. Hence the velocity and temperature rises in the entire boundary layer. However, the mass concentration is to reduce marginally with increase in Ec (fig.13). Higher the dissipation larger the skin friction and Sherwood number, smaller the Nusselt Number (table.3) The effect of Soret and Dufour effects on the velocity, temperature and concentration is exhibited in figs.14-16. From fig.14 that the velocity profiles experience an enhancement with increasing values of Sr (or decreasing Dufour parameter

(Du). This is due to the fact that an increase in Sr (or decrease in Du) increases the thickness of the boundary layer. From fig.15 we find that an increase in $Sr \leq 1.0$ (or $Du \leq 0.04$) increases the temperature and for higher $Sr \geq 1.5$ (or $Du \geq 0.05$) we notice a decrease in temperature in the thermal boundary layer. An increase in Sr (or decrease in Du) enhances the concentration. This may be attributed to the fact that the thickness of the solutal boundary layer increases with increase in Sr which in turn rises the concentration in the flow region (fig.16). Increasing Sr (or decreasing Du) increases the skin friction, Nusselt number and Sherwood number at the wall (table.3)

Figs.17-19 represent the influence of thermophoresis on velocity, temperature and concentration. It is observed from the fig.12a that increasing the values of thermophoresis parameter τ increases the thickness of the momentum boundary layer while an increase in τ , reduces the thickness of the thermal and solutal boundary layers (figs.18-19). Increase in thermophoretic parameter (\square) reduces the Skin friction, Nusselt number and increases Sherwood number at the wall (table.3).

CONCLUSIONS

The combined effects of suction and thermophoresis on the mixed convective heat and mass transfer of a fluid through a porous medium in the presence of Thermo-Diffusion and Diffusion-Thermo effects were investigated. Similarity transformations are used to transform the resulting partial differential equations into ordinary differential equations and numerically solved by using the Finite element method. It is observed from the graphs that increasing Soret parameter Sr (or Dufour parameter Du) larger the velocity, concentration, Nusselt number and smaller the Skin friction and Sherwood number. Higher the radiative heat flux larger the velocity, temperature, Skin friction and Sherwood number and smaller the concentration and Nusselt Number. Higher the dissipation larger the velocity, temperature and smaller the concentration. The Skin friction and Sherwood number increases and Nusselt number. Increasing the values of thermophoresis parameter τ increases the thickness of the momentum boundary layer, Sherwood number at the wall, reduces the thickness of the thermal and solutal boundary layers, the Skin

friction, Nusselt number at the wall. Excellent agreement of the present results with those of Loganatham (13) with $Sr = Du = Ec = a_1 = B_1 = 0$ was obtained.

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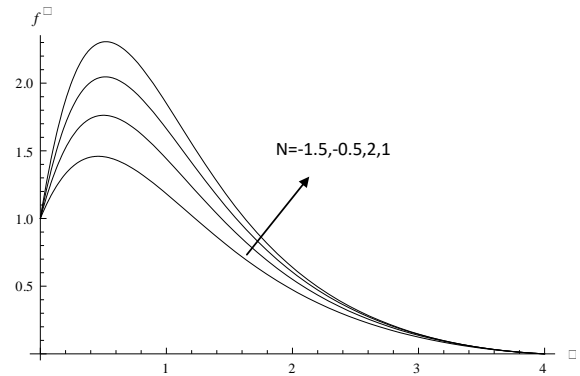


Fig.2: Variation of f' with N
 $G=2, M=0.5, D^+ = 0.1, Sc=1.3, Rd=0.5, A_1, B_1=0.1, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

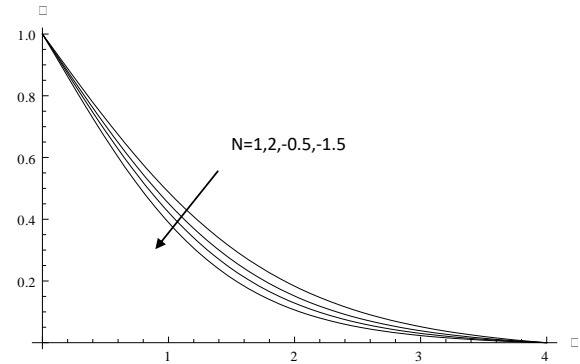


Fig. 2 : Variation of θ with N
 $G=2, M=0.5, D^+ = 0.1, Sc=1.3, Rd=0.5, A_1, B_1=0.1, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

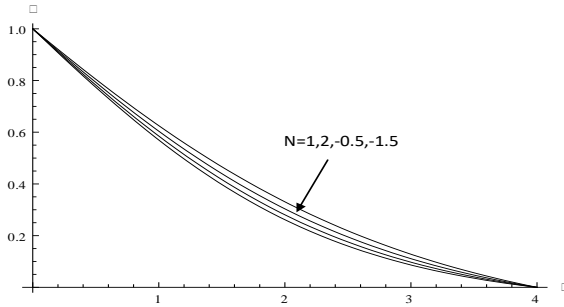


Fig.3 : Variation of ϕ with N
 $G=2, M=0.5, D^1=0.1, Sc=1.3, Rd=0.5, A_1B_1=0.1, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

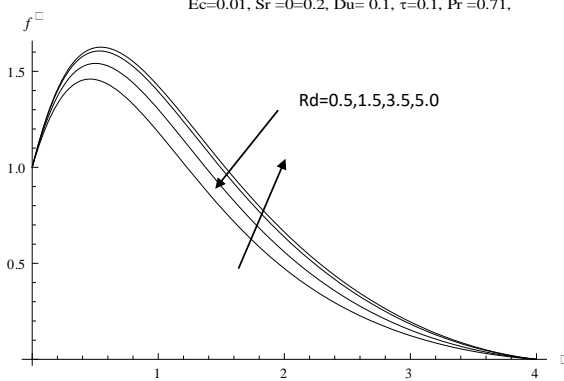


Fig.5 : Variation of f' with Rd
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, A_1B_1=0.1, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

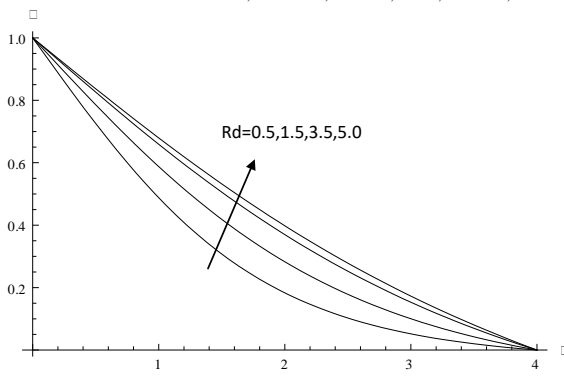


Fig.6 : Variation of θ with Rd
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, A_1B_1=0.1, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

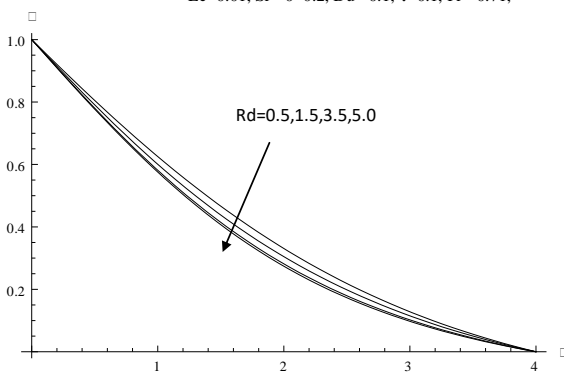


Fig.7 : Variation of ϕ with Rd
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, A_1B_1=0.1, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

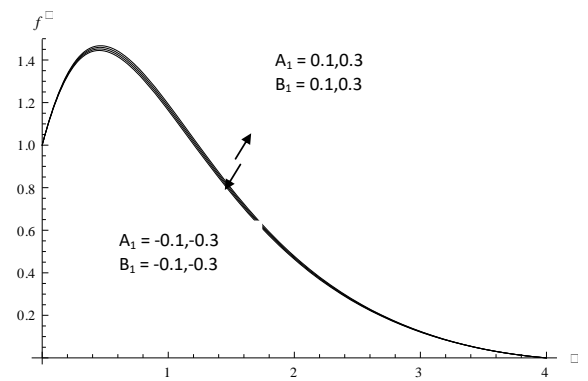


Fig.8 : Variation of f' with A_1B_1
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

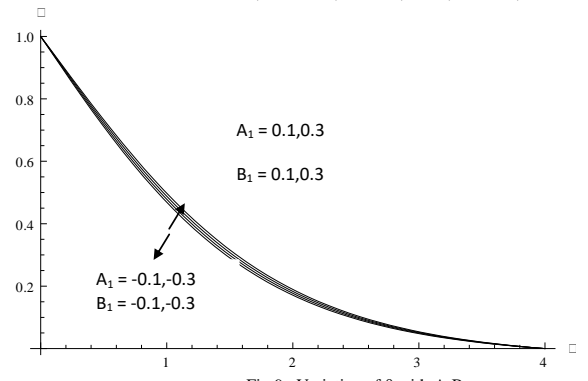


Fig.9 : Variation of θ with A_1B_1
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

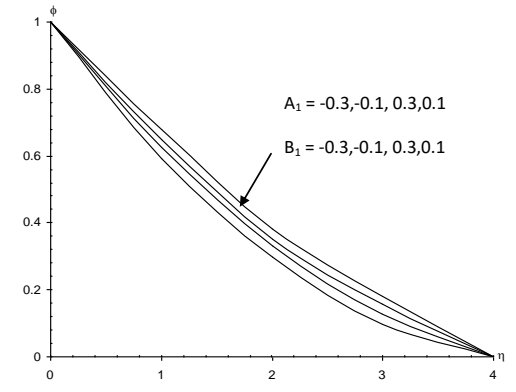


Fig.10 : Variation of ϕ with A_1B_1
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, f_w=0.2,$
 $Ec=0.01, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

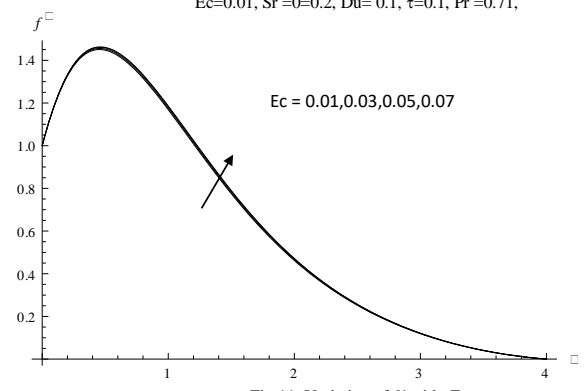


Fig.11 : Variation of f' with Ec
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

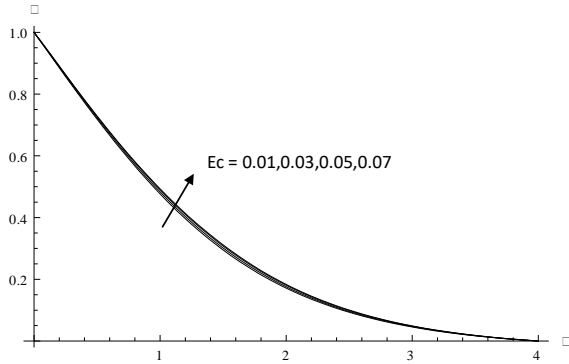


Fig.12 : Variation of θ with Ec
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

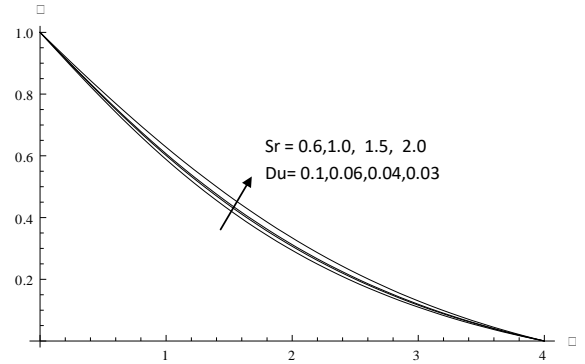


Fig.16 : Variation of ϕ with Sr & Du
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Ec=0.01, \tau=0.1, Pr=0.71,$

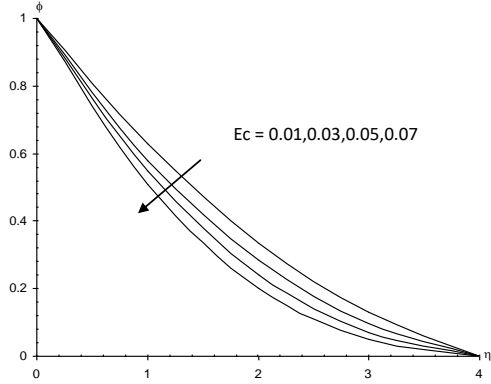


Fig.13 : Variation of ϕ with Ec
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Sr=0=0.2, Du=0.1, \tau=0.1, Pr=0.71,$

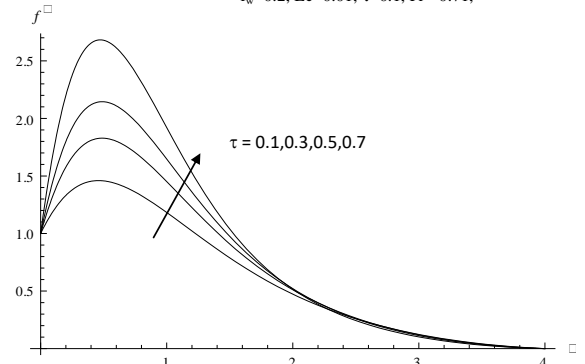


Fig.17 : Variation of f' with τ
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Ec=0.01, Sr=0=0.2, Du=0.1, Pr=0.71,$

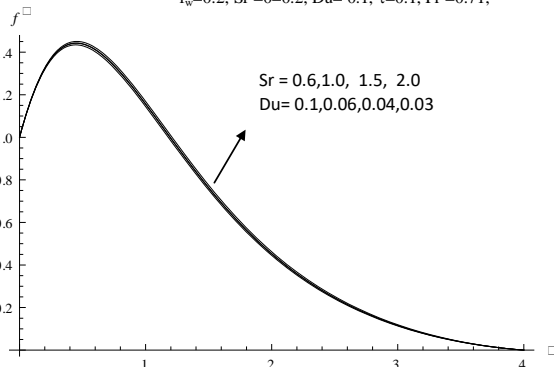


Fig.14 : Variation of f' with Sr & Du
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Ec=0.01, \tau=0.1, Pr=0.71,$

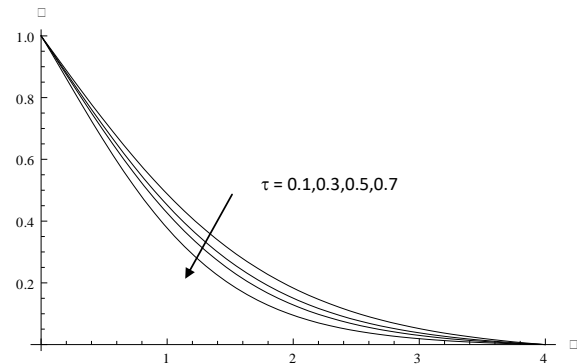


Fig.18 : Variation of θ with τ
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Ec=0.01, Sr=0=0.2, Du=0.1, Pr=0.71,$

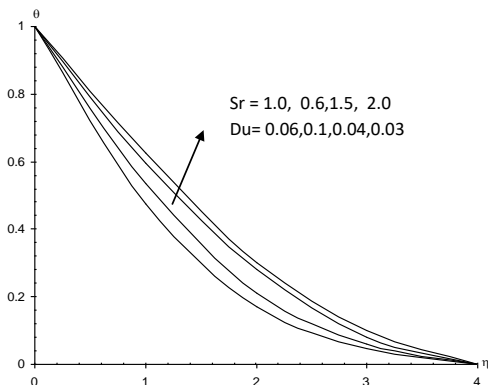


Fig.15 : Variation of θ with Sr & Du
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1, f_w=0.2,$
 $Ec=0.01, \tau=0.1, Pr=0.71,$

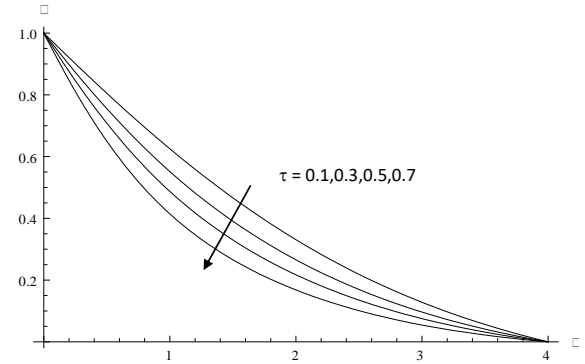


Fig.19 : Variation of ϕ with τ
 $G=2, M=0.5, D^1=0.1, N=1, Sc=1.3, Rd=0.5, A_1B_1=0.1,$
 $f_w=0.2, Ec=0.01, Sr=0=0.2, Du=0.1, Pr=0.71,$

Table 3 : Shear stress, Nusselt number and Sherwood number at $\eta=0$

Parameter	$\tau(0)$	Nu(0)	Sh(0)	
G	2	1.05356	0.808618	0.363286
	5	2.93674	0.907236	0.397023
	10	4.55613	0.963948	0.422464
	15	6.02521	0.999544	0.443391
M	0.5	3.77003	0.93902	0.41046
	1.0	2.16806	0.85318	0.37663
	1.5	0.02734	1.08419	2.97192
	2.0	0.01308	1.11337	5.53469
N	1.0	3.77003	0.93902	0.410465
	2.0	7.92865	1.03847	0.478077
	-0.5	-1.09201	0.49294	0.303574
	-1.5	-3.25348	0.27484	0.346899
Sc	0.24	3.77003	0.939025	0.410465
	0.66	3.31062	0.890686	0.477196
	1.3	3.13542	0.877389	0.503599
	2.01	-0.14614	7.302526	3.728084
D ⁻¹	0.2	4.43433	0.939026	0.410465
	0.4	4.38749	0.961547	0.421435
	0.6	4.28252	0.965825	0.423659
	0.8	3.77003	0.967696	0.424649
Ec	0.01	3.77003	0.939026	0.410465
	0.03	3.98326	0.843233	0.415989
	0.05	4.16956	0.773494	0.42087
	0.07	4.28471	0.735579	0.423917
Pr	0.71	3.77003	0.93902	0.410465
	1.71	3.10242	1.35002	0.394785
	3.71	2.87664	1.63266	0.389989
	7.0	2.75859	1.86357	0.387505
τ	0.5	3.77003	0.93902	0.410465
	1.0	3.17667	0.881345	0.521189
	1.5	2.80295	0.842299	0.633256
	2.0	2.35754	0.791708	0.872281
Sr/Du	2/0.03	3.77003	0.93902	0.410465
	1.5/0.04	3.728947	0.858983	0.404471
	1/0.06	3.70455	0.838078	0.463936
	0.6/0.1	3.92295	0.744862	0.494614
A ₁₁ , B ₁₁		3.77003	0.93902	0.410465
		3.8519	0.905269	0.412607
		3.69023	0.974368	0.408377
		3.40821	1.13327	0.401316
Rd		3.77003	0.93902	0.410465
		5.22673	0.594176	0.455042
		6.80058	0.434309	0.514608
		7.50711	0.389	0.543635
A		3.77003	0.93902	0.410465
		3.77003	0.93902	0.410465
		3.77003	0.93902	0.410465
		3.77003	0.93902	0.410465